Field theory approach to one-dimensional electronic systems *

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Abstract

In this talk I describe a field-theoretical approach recently introduced in refs.[1]-[3], that can be used as an alternative framework to study one-dimensional systems of highly correlated particles.

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Contents

1 Motivations

In this talk I describe recent work on the bosonization of a non-local Quantum Field Theory (QFT) and its application to the study of one-dimensional (1d) many-body systems. This research has been done in collaboration with **Virginia Manías**, **Marta Trobo** and **Cecilia von Reichenbach** from the University of La Plata, **Daniel Barci** from Universidade do Estado do Rio de Janeiro (UERJ), Brasil, and **Kang Li** from Hangzhou University, China.

Since Mattis and Lieb [5] showed how to correctly quantize the model of 1d fermions proposed by Luttinger [6] (which was nothing but a slightly modified version of the model introduced by Tomonaga many years before [7]), the study of the highly-correlated electronic liquid remained an outstanding problem in mathematical physics. Perhaps the main reason for this sustained interest has been the rather strange behavior of the 2-point fermionic correlator which, in 1d presents a branch cut instead of a pole structure. Consequently one cannot define one single particle states , meaning that the usual quasiparticle (Landau) picture fails. One then expects that only collective modes (CDW4s and SDW4s) will be present in the spectrum. These features, together with the separation of spin and density waves and the disappearence of the Fermi edge in the momentum distribution, characterize the so called Luttinger liquid behavior [8]. During the last decade a more practical and surprising motivation has been added to the previous academic reasons to investigate the 1d electronic system. Indeed, striking developments in the field of nanofabrication have allowed to build real 1d semiconductors [9] (Please see [10] for a review of recent theoretical developments). Stimulated by this curious situation in which simplified and unrealistic models became closely related to reality, we tried to employ some path-integral techniques, which have been very useful in the study of $1+1$ QFT4s, in order to provide an alternative field-theoretical approach to the study of 1d many-body systems. This led us to propose a non-local generalization of the Thirring model which contains the Tomonaga-Luttinger model as a particular case. In the next section we shall show how to deal with such a non-local theory.

2 The Non-Local Thirring model

We start by considering the Euclidean vacuum functional

$$Z = N \int D\bar{\Psi} D\Psi \; e^{-S} \tag{1}$$

where N is a normalization constant and S is given by

$$S = \int d^2x \; \bar{\Psi} i\partial\!\!\!/ \Psi - \frac{g^2}{2} \int d^2x d^2y \; [V_{(0)}(x,y) J_0(x) J_0(y) + V_{(1)}(x,y) J_1(x) J_1(y)] \tag{2}$$

where $J_\mu = \bar{\Psi}\gamma_\mu\Psi$ and $V_{(\mu)}(x,y)$ is an arbitrary function of two variables. Note that for $V_{(0)} = V_{(1)} = \delta^2(x-y)$ one recovers the usual covariant Thirring model.

As it is habitual in the path-integral approach to the usual Thirring model, one eliminates this quartic fermionic interaction by introducing an auxiliary vector field A_μ. As we shall see, in the present non-local case one needs one more auxiliary field to achieve the same goal. In order to depict this procedure we first observe that S can be splitted in the form

$$S = S_0 + S_{int} \tag{3}$$

where

$$S_0 = \int d^2x \; \bar{\Psi} i\partial\!\!\!/ \Psi, \tag{4}$$

and

$$S_{int} = -\frac{g^2}{2} \int d^2x \; J_\mu K_\mu. \tag{5}$$

In this last expression J_μ is the usual fermionic current,

$$J_\mu(x) = \bar{\Psi}(x)\gamma_\mu\Psi(x), \tag{6}$$

and K_μ is a new current defined as

$$K_\mu(x) = \int d^2y \; V_{(\mu)}(x,y) J_\mu(y). \tag{7}$$

Please note that no sum over repeated indices is implied when a subindex (μ) is involved. Introducing a functional delta the partition function can be expressed as

$$Z = N \int D\bar{\Psi} D\Psi D\tilde{A}_\mu D\tilde{B}_\mu \; exp[-\{S_0 + \int d^2x [\tilde{A}_\mu \tilde{B}_\mu - \frac{g}{\sqrt{2}}(\tilde{A}_\mu J_\mu + \tilde{B}_\mu K_\mu)]\}] \tag{8}$$

On the other hand, the fermionic piece of the action can be written as

$$S_0 - \frac{g}{\sqrt{2}} \int d^2x \; (\tilde{A}_\mu J_\mu + \tilde{B}_\mu K_\mu) = \int d^2x \; \bar{\Psi}[i\partial\!\!\!/ - \frac{g}{\sqrt{2}} \gamma_\mu (\tilde{A}_\mu + \bar{B}_\mu)]\Psi, \tag{9}$$

where we have defined

$$\bar{B}_\mu(x) = \int d^2y \; V_{(\mu)}(y,x) \tilde{B}_\mu(y). \tag{10}$$

For later convenience we shall invert (10) in the form

$$\tilde{B}_\mu(x) = \int d^2y \; b_{(\mu)}(y,x) \bar{B}_\mu(y), \tag{11}$$

with $b_{(\mu)}(y,x)$ satisfying

$$\int d^2y \; b_{(\mu)}(y,x) V_{(\mu)}(z,y) = \delta^2(x-z). \tag{12}$$

Equation (9) suggests the following change of auxiliary variables:

$$\frac{1}{\sqrt{2}}(\tilde{A}_\mu + \bar{B}_\mu) = A_\mu, \tag{13}$$

$$\frac{1}{\sqrt{2}}(\tilde{A}_\mu - \bar{B}_\mu) = B_\mu. \tag{14}$$

From now on we shall restrict our study to the case in which the bilocal functions $V_{(\mu)}$ and $b_{(\mu)}$ are symmetric. Under these conditions the partition function of the system is given by

$$Z = N_1 \int DA_\mu DB_\mu \; det(i\partial\!\!\!/ + gA\!\!\!/) e^{-S[A,B]}, \tag{15}$$

where $S[A,B]$ is such that A and B are decoupled. Moreover, B is not coupled to fermionic fields either, and can then be absorbed in the normalization

constant. (Besides, B has negative metric and must be eliminated in order to have a good Hilbert space. The appearence of this ghost is not due to non-locality, it is already present in the local model [11]). Thus we have been able to express Z in terms of a fermionic determinant:

$$Z = N_2 \int DA_\mu det(i\partial\!\!\!/ + gA\!\!\!/) e^{-S[A]}, \tag{16}$$

where N_2 includes the contribution of the "non-local ghost" B_μ and $S[A]$ will be rewritten in terms of two new scalars Φ and η by using

$$A_\mu(x) = \epsilon_{\mu\nu}\partial_\nu\Phi(x) + \partial_\mu\eta(x). \tag{17}$$

At this stage one can employ the machinery of the path-integral approach to bosonization, based on a chiral change in the fermionic path-integral measure with Φ and η as parameters. Taking into account the corresponding Jacobian we finally get

$$Z = N \int D\Phi D\eta \; e^{-S_{eff}} \tag{18}$$

where

$$\begin{aligned} S_{eff} &= \frac{g^2}{2\pi}\int d^2x \; (\partial_\mu\Phi)^2 + \\ &+ \frac{1}{2}\int d^2x d^2y [b_{(0)}(y,x)\partial_1\Phi(x)\partial_1\Phi(y) + b_{(1)}(y,x)\partial_0\Phi(x)\partial_0\Phi(y)] + \\ &+ \frac{1}{2}\int d^2x d^2y [b_{(0)}(y,x)\partial_0\eta(x)\partial_0\eta(y) + b_{(1)}(y,x)\partial_1\eta(x)\partial_1\eta(y)] + \\ &+ \int d^2x d^2y [b_{(0)}(y,x)\partial_0\eta(x)\partial_1\Phi(y) - b_{(1)}(y,x)\partial_1\eta(x)\partial_0\Phi(y)] \end{aligned} \tag{19}$$

Equations (18) and (19) constitute the main result of this Section. Thus, we have been able to extend the path-integral approach to bosonization, previously applied to the solution of local QFT's, to a Thirring-like model of fermions with a non-local interaction term. More specifically, we have shown the equivalence between the fermionic partition function (1) and the functional integral (18) corresponding to the two bosonic degrees of freedom Φ and η with dynamics governed by (19). The contribution to this action coming from the fermionic Jacobian (the first term in the r.h.s of (19)) exactly

coincides with the one which is obtained in the local case. On the other hand, the effect of non-locality is contained in the remaining terms, through the inverse potentials $b_\mu(x,y)$. Note that, even in the non-local case, Φ and η become decoupled for $b_{(0)} = b_{(1)}$. Of course, when $b_{(0)} = b_{(1)} = \delta^2(x-y)$, one recovers the bosonic version of the local Thirring model.

The spectrum of this bosonic model can be more easily analyzed in momentum space. Indeed, by Fourier transforming (19) one obtains

$$\begin{aligned} S_{eff} &= \frac{1}{(2\pi)^2}\int d^2p\{\hat{\Phi}(p)\hat{\Phi}(-p)A(p) \\ &+ \hat{\eta}(p)\hat{\eta}(-p)B(p) + \hat{\Phi}(p)\hat{\eta}(-p)C(p)\}, \end{aligned} \tag{20}$$

where

$$A(p) = \frac{g^2}{2\pi}\ p^2 + \frac{1}{2}[\hat{b}_{(0)}(p)p_1^2 + \hat{b}_{(1)}(p)p_0^2], \tag{21}$$

$$B(p) = \frac{1}{2}[\hat{b}_{(0)}(p)p_0^2 + \hat{b}_{(1)}(p)p_1^2], \tag{22}$$

$$C(p) = [\hat{b}_{(0)}(p) - \hat{b}_{(1)}(p)]p_0p_1, \tag{23}$$

and $\hat{\Phi}, \hat{\eta}$ and $\hat{b}_{(\mu)}$ are the Fourier transforms of Φ, η and $b_{(\mu)}$ respectively. Eq. (20) can be easily diagonalized through the change

$$\begin{aligned} \hat{\Phi} &= \hat{\zeta} - \frac{C}{2A}\hat{\xi} \\ \hat{\eta} &= \hat{\xi}. \end{aligned} \tag{24}$$

We then have the following propagators for $\hat{\zeta}$ and $\hat{\xi}$:

$$G_{\zeta}^{-1}(p) = \lambda p^2 + \frac{1}{2}[\hat{b}_{(0)}p_1^2 + \hat{b}_{(1)}p_0^2] \tag{25}$$

$$G_{\xi}^{-1}(p) = \frac{\lambda p^2[\hat{b}_{(0)}p_0^2 + \hat{b}_{(1)}p_1^2] + \frac{\hat{b}_{(0)}\hat{b}_{(1)}}{2}p^4}{2\lambda p^2 + \hat{b}_{(0)}p_1^2 + \hat{b}_{(1)}p_0^2}, \tag{26}$$

where $\lambda = \frac{g^2}{2\pi}$ and $\hat{b}_{(0)}$, $\hat{b}_{(1)}$ are functions of p. These expressions are further simplified in the case $\hat{b}_{(0)} = \hat{b}_{(1)}$. In particular, when $\hat{b}_{(0)} = \hat{b}_{(1)} \propto \frac{1}{p^2}$ the $\hat{\zeta}$ field acquires a mass, whereas $\hat{\xi}$ becomes a non propagating field.

3 The Tomonaga-Luttinger model

The approach depicted above will be now applied to the TL model [5] [6] [7]. This model describes a non-relativistic gas of spinless particles (electrons) in which the free dispersion relation is taken to be linear. The free-particle Hamiltonian is given by

$$H_0 = v_F \int dx \Psi^\dagger(x)(\sigma_3 p - p_F)\Psi(x) \tag{27}$$

where v_F and p_F are the Fermi velocity and momentum respectively ($v_F p_F$ is a convenient origin for the energy scale). σ_3 is a Pauli matrix and Ψ is a column bispinor with components Ψ_1 and Ψ_2 ($\Psi^\dagger = (\Psi_1^\dagger \quad \Psi_2^\dagger)$). The function $\Psi_1(x)$ [$\Psi_2(x)$] is associated with the motion of particles in the positive [negative] x direction. The interaction piece of the Hamiltonian, when only forward scattering is considered, is

$$H_{int} = \int dx \int dy \sum_{a,b} \Psi_a^\dagger(x)\Psi_a(x)V_{ab}(x,y)\Psi_b^\dagger(y)\Psi_b(y) \tag{28}$$

where $a, b = 1, 2$, and the interaction matrix is parametrized in the form

$$V_{ab} = \begin{pmatrix} v_1 & v_2 \\ v_2 & v_1 \end{pmatrix}. \tag{29}$$

Using the imaginary-time formalism one can show that the finite-temperature [12] [13] action for this problem becomes

$$\begin{aligned} S_{TL} &= \int_0^\beta d\tau \int dx \, \{\bar{\Psi}\gamma_0(\partial_\tau - v_p p_F)\Psi + v_F \bar{\Psi}\gamma_1 \partial_x \Psi\} \\ &+ \int_0^\beta d\tau \int dx \int dy \sum_{a,b} \Psi_a^\dagger \Psi_a(x,\tau) V_{ab}(x,y) \Psi_b^\dagger \Psi_b(y,\tau). \end{aligned} \tag{30}$$

For simplicity, in this Section we shall set $v_F = 1$ and consider the case $v_1 = v_2$ in (29) [5]. We shall also restrict ourselves to the zero temperature limit ($\beta \to \infty$). Under these conditions it is easy to verify that S_{TL} coincides with the non-local Thirring model discussed in the precedent Section, provided that the following identities hold:

$$
\begin{aligned}
g^2 &= 2 \\
V_{(0)}(x,y) &= v_1(x,y) = v_2(x,y) = v(x_1 - y_1)\delta(x_0 - y_0) \\
V_{(1)} &= 0
\end{aligned}
\tag{31}
$$

Of course one has also to make the shift $\bar{\Psi}\gamma_0\partial_0\Psi \rightarrow \bar{\Psi}\gamma_0(\partial_0 - p_F)\Psi$ and identify $x_0 = \tau$, $x_1 = x$.

One then can employ the method described in the precedent Section in order to study the Tomonaga-Luttinger model. This model has been previously studied, through a different functional approach, by D.K. Lee and Y. Chen [14]. These authors, however, avoided the use of the decoupling technique presented here. Our approach is particularly useful when considering spin-flipping interactions, i.e. the non-Abelian extension of the model. For simplicity I will not consider this case in this work, but the interested reader will find related discussions in refs. [1] and [2].

Let us first focus our attention to the dispersion relations corresponding to the elementary excitations of the model at hand. These states correspond to the normal modes whose dynamics is governed by the action (20). As it is well-known, the spectrum of these modes is obtained from the poles of the corresponding propagators. Alternatively, one can write the effective Lagrangian as

$$
L_{eff} = \frac{1}{(2\pi)^2} \begin{pmatrix} \hat{\Phi} & \hat{\eta} \end{pmatrix} \begin{pmatrix} A & C/2 \\ C/2 & B \end{pmatrix} \begin{pmatrix} \hat{\Phi} \\ \hat{\eta} \end{pmatrix} \tag{32}
$$

(with A, B and C defined in (21)-(23)) and solve the equation

$$
\Delta(p) = 0, \tag{33}
$$

with $\Delta(p) = C^2(p) - 4A(p)B(p)$. Going back to real frecuencies : $p_0 = i\omega$, $p_1 = q$, (33) yields a biquadratic equation for ω. The relevant solution is

$$
\omega_-^2(q) = \frac{\hat{b}_{(1)}}{\hat{b}_{(0)}} \frac{2\lambda + \hat{b}_{(0)}}{2\lambda + \hat{b}_{(1)}} q^2. \tag{34}
$$

Inserting now the identities (31) in (34) ($\lambda = \frac{g^2}{2\pi}$) we obtain

$$\omega_-^2(q) = q^2\{1 + \frac{2v(q)}{\pi}\} \tag{35}$$

which is the well-known result for the spectrum of the charge-density excitations of the TL model in the Mattis-Lieb version [5].

The next step is to compute the electron propagator. To this end, having established the correspondence between the TL and NLT models, we can use the decoupling technique at the level of the 2-point function. As usual, the non-vanishing components of the fermionic 2-point function are factorized into fermionic and bosonic contributions. Carefully taking into account the Fermi momentum in the free-fermion factor one gets

$$G_\pm^0(z) = \frac{e^{\pm ip_F z_1}(z_0 \pm iz_1)}{2\pi \mid z \mid^2} \tag{36}$$

whereas the bosonic factor becomes

$$B_\pm(z) = exp[\frac{1}{\pi^2}\int d^2p\ \frac{v(p)}{p^2}\frac{sen^2(\frac{p.z}{2})(p_0 \pm ip_1)^2}{p_0^2 + (1+2v/\pi)p_1^2}]. \tag{37}$$

The momentum distribution for branch 1 (2) electrons is given by

$$N_{\frac{1}{2}}(p_1) = \mathtt{C}(\Lambda)\int_{-\infty}^{\infty} dz_1\ e^{-ip_1 z_1}\lim_{z_0\to 0} G_\pm(z_0, z_1) \tag{38}$$

Replacing (36) and (37) in (38) we get

$$\begin{aligned} N_{\frac{1}{2}}(p_1) &= \pm\frac{\mathtt{C}(\Lambda)i}{2\pi}\int_{-\infty}^{\infty} dz_1\ e^{-iz_1(p_1 \mp p_F)} \times \\ &\times \frac{1}{z_1} exp\{\frac{1}{\pi^2}\int \frac{d^2p\ v(p)}{p^2} sen^2(\frac{p_1 z_1}{2})\frac{(p_0^2 - p_1^2)}{p_0^2 + [1+2v(p)/\pi]p_1^2}\}. \end{aligned} \tag{39}$$

Here $\mathtt{C}(\Lambda)$ is a normalization constant depending on an ultraviolet cutoff Λ. In the local limit, in which $v(p) = const$, the integrals in the momentum can be easily evaluated and one obtains

$$N_{\frac{1}{2}}(p_1) = \pm\frac{i}{2\pi}\mathtt{C}(\Lambda)\int_{-\infty}^{\infty} dz_1 \frac{e^{-i(p_1 \mp p_F)z_1}}{z_1^{1+\sigma}} \tag{40}$$

with

$$\sigma = \frac{1}{2}\{(1+\frac{2v}{\pi})^{1/2} + (1+\frac{2v}{\pi})^{-1/2} - 2\} \tag{41}$$

Note that in the free case $v \to 0$ one gets $\sigma = 0$, which leads to the well-known normal Fermi-liquid behavior,

$$N_{\substack{1\\2}} \propto \theta(p_1 \pm p_F). \tag{42}$$

As soon as the interaction is switched on, one has $\sigma \neq 0$ and the Fermi edge singularity is washed out, giving rise to the so called Luttinger-liquid behavior [8]. It has been emphasized recently [15] that the experimental data obtained for one-dimensional structures can be succesfully explained on the basis of standard Fermi-liquid theory. We believe that our approach could be useful to explore some modifications of the TL model to take into account, for instance, the presence of impurities or defects, that might yield a restoration of the edge singularity. The issue of how to incorporate impurities in our framework will be briefly addressed in the next Section.

4 Fermionic impurities

The main purpose of this Section is to comment on the extension of the path-integral approach to non-local bosonization proposed in [1], to the case in which an interaction between the electrons and a finite density of fermionic impurities is included in the action. This generalization of the non-local bosonization procedure provides a new way to examine the low-energy physics of the TL model in the presence of localized impurities, that could allow to make contact with recent very interesting studies [16],[17] on the response of a Luttinger liquid to localized perturbations.

We describe the impurities following the work of Andrei [18], who introduced a new fermionic field with vanishing kinetic energy to represent a finite density of impurities, arbitrarily (not randomly) situated. This treatment has been previously employed, for example, in the path-integral bosonization of the Kondo problem [19].

We introduce a non-local diagonal potential matrix binding impurities and electrons through their corresponding fermionic currents. This procedure allows to treat a wide range of possible interactions, depending on the precise functional form of the potential matrices. The complete coupling term includes interactions between charge, current, spin and spin-current densities. Our functional approach enables us to obtain an effective action governing the dynamics of the collective modes, providing then a practical framework to face a non-perturbative analysis of bosonic degrees of freedom in the presence of impurities.

We start from the partition function

$$Z = \int D\bar{\Psi}\ D\Psi\ D\bar{d}\ Dd\ e^{-S}, \tag{43}$$

where the action S can be splitted as

$$S = S_0 + S_{int}, \tag{44}$$

with

$$S_0 = \int d^2x\ [\bar{\Psi} i\not{\partial}\Psi + d^\dagger i\partial_t d] \tag{45}$$

and

$$S_{int} = -\int d^2x\ d^2y\ [J^a_\mu(x) V^{ab}_{(\mu)}(x,y) J^b_\mu(y) + J^a_\mu(x) U^{ab}_{(\mu)}(x,y) S^b_\mu(y)], \tag{46}$$

where the electron field Ψ is written as

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix},$$

with Ψ_1 (Ψ_2) in the fundamental representation of U(N), describing right (left) movers, whereas the impurity field d is given by

$$d = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}.$$

Note the absence of a spatial derivative in the free piece of the impurity action, meaning that the corresponding kinetic energy is zero. Concerning the electronic kinetic energy, we have set the Fermi velocity equal to 1.
The interaction pieces of the action have been written in terms of U(N) currents J_μ^a and S_μ^a, defined as

$$\begin{aligned} J_\mu^a &= \bar{\Psi}\gamma_\mu\lambda^a\Psi, \\ S_\mu^a &= \bar{d}\gamma_\mu\lambda^a d, \quad a = 0, 1, ..., N^2-1, \end{aligned} \tag{47}$$

with $\lambda^0 = I/2$, $\lambda^j = t^j$, t^j being the SU(N) generators normalized according to $tr(t^i t^j) = \delta^{ij}/2$. $V_{(\mu)}^{ab}(x,y)$ and $U_{(\mu)}^{ab}(x,y)$ are $N^2 \times N^2$ matrices whose elements are symmetric bilocal arbitrary potentials describing the electron-electron (e-e) and the electron-impurity (e-i) interactions, respectively.

Although we have obtained a bosonized effective action for the general (non-abelian, spin-flipping) problem (See [2] for details), here I shall restrict myself to the maximal abelian subgroup of U(2). In this case the model describes a many-body system of spin-$\frac{1}{2}$ fermions when spin-flipping processes are not allowed. Now, the potential matrices are diagonal whose elements can be written in terms of the g-functions defined by Sólyom [20] as

$$\begin{aligned} V_{(0)}^{00} &= \frac{1}{4}(g_{4\parallel} + g_{4\perp} + g_{2\parallel} + g_{2\perp}), \\ V_{(0)}^{11} &= \frac{1}{4}(g_{4\parallel} - g_{4\perp} + g_{2\parallel} - g_{2\perp}), \\ V_{(1)}^{00} &= \frac{1}{4}(-g_{4\parallel} - g_{4\perp} + g_{2\parallel} + g_{2\perp}), \\ V_{(1)}^{11} &= \frac{1}{4}(-g_{4\parallel} + g_{4\perp} + g_{2\parallel} - g_{2\perp}). \end{aligned} \tag{48}$$

It is straightforward to verify that the e-e interaction term in (46) contains the whole set of diagrams associated to forward scattering processes without spin-flips. Let us recall that the coupling constants for incident fermions with parallel spins are denoted by the susbscript $\|$ and that for fermions with opposite spins by the subscript $\perp$. In the g_2 processes the two branches (left and right moving particles) are coupled, while in the g_4 processes all four participating fermions belong to the same branch. The Tomonaga-Luttinger model, with charge-density fluctuations only, corresponds to $V^{11}_{(0)} = V^{11}_{(1)} = 0$. In a completely analogous way we introduce the potentials that couple electron and impurity currents in the form

$$\begin{aligned} U^{00}_{(0)} &= \frac{1}{4}(h_{4\|} + h_{4\perp} + h_{2\|} + h_{2\perp}), \\ U^{11}_{(0)} &= \frac{1}{4}(h_{4\|} - h_{4\perp} + h_{2\|} - h_{2\perp}), \\ U^{00}_{(1)} &= \frac{1}{4}(-h_{4\|} - h_{4\perp} + h_{2\|} + h_{2\perp}), \\ U^{11}_{(1)} &= \frac{1}{4}(-h_{4\|} + h_{4\perp} + h_{2\|} - h_{2\perp}). \end{aligned}$$

This description includes both charge and spin density interactions, as well as spin-current interactions. A Kondo-like interaction, i.e. the coupling between spin densities only, corresponds to the case $U^{00}_{(0)} = U^{00}_{(1)} = 0$.

In order to carry out the bosonization of the model one has to face some technical difficulties that we shall not describe here (Again, see [2] for details). Let me say that, in this case one has two fermionic determinants in the partition function, one associated to electrons and a new one, related to the impurity degrees of freedom. When they are conveniently decoupled, an effective action for the collective modes of the system is again obtained. In other words, one is left with a partition function in terms of bosonic fields Φ^i and η^i which, by comparison with the impurity-free case, one naturally identifies with the collective modes of the system. The result is

$$Z = \int D\Phi^i \; D\eta^i \; exp - \left\{ S^{00}_{eff} + S^{11}_{eff} \right\}, \tag{49}$$

where the actions, in Fourier space, are given by

$$
\begin{aligned}
S^{ii}_{eff} &= \frac{1}{(2\pi)^2}\int d^2p[\hat{\Phi}^i(p)A^{ii}(p)\hat{\Phi}^i(-p)+\hat{\eta}^i(p)B^{ii}(p)\hat{\eta}^i(-p)+ \\
&+ \hat{\Phi}^i(p)\frac{C^{ii}(p)}{2}\hat{\eta}^i(-p)+\hat{\eta}^i(p)\frac{C^{ii}(p)}{2}\hat{\Phi}^i(-p)], \qquad (50)
\end{aligned}
$$

where

$$
\begin{aligned}
A(p) &= \frac{1}{\Delta(p)}\left\{\frac{p^2}{\pi}\Delta - a_0a_1\frac{p_1^2}{\pi}+\frac{1}{2\pi}(a_0^2p_1^2-a_1^2p_0^2)-2a_0^2a_1^2(\frac{p_1^2}{b_1}+\frac{p_0^2}{b_0})\right\}, \\
B(p) &= \frac{1}{\Delta(p)}\left\{\frac{p_1^2}{\pi}a_0a_1+\frac{1}{2\pi}(a_0^2p_0^2-a_1^2p_1^2)-2a_0^2a_1^2(\frac{p_1^2}{b_0}+\frac{p_0^2}{b_1})\right\}, \\
C(p) &= \frac{1}{\Delta(p)}\left\{\frac{a_0a_1}{\pi}(\frac{p_1^3}{p_0}-p_0p_1)+\frac{p_0p_1}{\pi}(a_0^2+a_1^2)+4p_0p_1a_0^2a_1^2(\frac{1}{b_0}-\frac{1}{b_1})\right\}, \\
\Delta(p) &= \frac{p_1^2}{4\pi^2p_0^2}+4(\frac{1}{4\pi}-\frac{a_1^2}{b_1})(\frac{1}{4\pi}+\frac{a_0^2}{b_0}). \qquad (51)
\end{aligned}
$$

For the sake of clarity we have omitted ii superindices in the above expressions, which are written in terms of the Fourier transforms of the inverse potentials. (Note that $b_\mu(p)=V^{-1}_{(\mu)}(p)$ and $a_\mu(p)=U^{-1}_{(\mu)}(p)$).

This is one of our main results. We have obtained a completely bosonized action for the collective modes corresponding to a system of electrons which interact not only between themselves, but also with fermionic localized impurities at $T=0$. This effective action describes the dynamics of charge density (Φ^0 and η^0) and spin density (Φ^1 and η^1)fields. As we can see, these modes remain decoupled as in the impurity free case. Their dispersion relations can be obtained from the poles of the corresponding propagators. Alternatively, one can write the effective Lagrangian as

$$
L^{ii}_{eff}=\frac{1}{2\pi}\left(\hat{\Phi}^i\ \hat{\eta}^i\right)\begin{pmatrix} A^{ii} & C^{ii}/2 \\ C^{ii}/2 & B^{ii}\end{pmatrix}\begin{pmatrix}\hat{\Phi}^i \\ \hat{\eta}^i\end{pmatrix}, \qquad (52)
$$

with A, B and C as defined above, and solve the equation

$$
C^2(p)-4A(p)B(p)=0. \qquad (53)
$$

Going to real frecuencies: $p_0=i\omega, p_1=q$, this equation has the following pair of relevant solutions:

$$
\omega^2_\rho(q)=q^2\frac{1+\frac{2}{\pi}V^{00}_{(0)}+\frac{1}{2\pi^2}\{(U^{00}_{(0)})^2-2U^{00}_{(0)}U^{00}_{(1)}\}}{1+\frac{2}{\pi}V^{00}_{(1)}-\frac{1}{2\pi^2}(U^{00}_{(1)})^2}, \qquad (54)
$$

$$\omega_\sigma^2(q) = q^2 \frac{1 + \frac{2}{\pi} V_{(0)}^{11} + \frac{1}{2\pi^2}\{(U_{(0)}^{11})^2 - 2U_{(0)}^{11}U_{(1)}^{11}\}}{1 + \frac{2}{\pi} V_{(1)}^{11} - \frac{1}{2\pi^2}(U_{(1)}^{11})^2}, \tag{55}$$

The first equation gives the dispersion relation associated to charge-density fluctuations $(\hat{\Phi}^0, \hat{\eta}^0)$, whereas the second one corresponds to spin-density modes $(\hat{\Phi}^1, \hat{\eta}^1)$.

As a confirmation of the validity of our approach, we note that the above dispersion relations, involving both e-e and e-i interaction potentials, coincide with the well-known result for the spectrum of charge and spin excitations in the TL model without impurities, obtained by choosing $V_{(1)} = U_{(0)} = U_{(1)} = 0$ and $V_{(0)} = v(q)$, in the above formulae.

Let us now consider the fermionic 2-point function

$$\langle \Psi(x)\bar{\Psi}(y)\rangle = \begin{pmatrix} o & G_1(x,y) \\ G_2(x,y) & 0 \end{pmatrix} \tag{56}$$

where

$$G_{1(2)}(x,y) = \begin{pmatrix} G_{1(2)\uparrow}(x,y) & 0 \\ 0 & G_{1(2)\downarrow}(x,y) \end{pmatrix} \tag{57}$$

The subindex 1(2) means that we consider electrons belonging to the branch 1(2), and $\uparrow$ ($\downarrow$) indicates that the field operator carries a spin up (down) quantum number. Let us recall that in the present case we have disregarded those processes with spin-flip. This is why the fermionic Green function do not have non-zero components with mixed spin indices.
To be specific we consider $G_{1\uparrow}$ (similar expressions are obtained for $G_{2\uparrow}$, $G_{1\downarrow}$ and $G_{2\downarrow}$). When the decoupling chiral change is performed, the components of the Green functions are factorized into fermionic and bosonic contributions in the form

$$\begin{aligned} G_{1\uparrow}(x,y) &= < \Psi_{1\uparrow}(x)\Psi_{1\uparrow}^\dagger(y) > \\ &= G_{1\uparrow}^{(0)}(x,y) < e^{\left\{[\Phi^0(y)-\Phi^0(x)]+i[\eta^0(y)-\eta^0(x)]\right\}} >_{00} \times \\ &\times < e^{\left\{[\Phi^1(y)-\Phi^1(x)]+i[\eta^1(y)-\eta^1(x)]\right\}} >_{11}, \end{aligned} \tag{58}$$

where $G_{1\uparrow}^{(0)}(x,y)$ is the free propagator, which involves the Fermi momentum p_F, and is given by

$$G_{1\uparrow}^{(0)}(x,y) = \frac{e^{ip_F z_1}}{2\pi \mid z \mid^2}(z_0 + iz_1). \tag{59}$$

The symbol $<>_{ii}$ means v.e.v. with respect to the action (50, 51). Exactly as we did in the impurity-free case ([1]), the bosonic factors in (58) can be evaluated by appropriately shifting the fields. Indeed, working in momentum-space, and defining the non-local operator

$$D(p;x,y) = e^{-ip.x} - e^{-ip.y}, \tag{60}$$

the functional integrations can be performed, yielding

$$\begin{aligned} < \Psi_{1\uparrow}(x)\Psi^{\dagger}_{1\uparrow}(y) >= G^{(0)}_{1\uparrow}(x,y) \qquad & exp\{-\int \frac{d^2p}{(2\pi)^2} D^2 \frac{A^{00} - B^{00} + iC^{00}}{4A^{00}B^{00} - (C^{00})^2}\} \\ & exp\{-\int \frac{d^2p}{(2\pi)^2} D^2 \frac{A^{11} - B^{11} + iC^{11}}{4A^{11}B^{11} - (C^{11})^2}\}, \end{aligned} \tag{61}$$

with A^{ii}, B^{ii} and C^{ii} given by (51). In order to continue the calculation one needs, of course, to specify the couplings and perform the integrals. This means that our formula could be used to test the effect of different e-e and e-i potentials on the behavior of the fermionic propagator.

As a final illustration of our procedure, now I will show how to compute the electronic momentum distribution, for a quite peculiar choice of e-e and e-i potential matrix elements.
Let us consider the momentum distribution of electrons belonging to branch 1 and with spin-up. This distribution is given by

$$N_{1\uparrow}(q) = C(\Lambda) \int_{-\infty}^{+\infty} dz_1 \; e^{-iqz_1} \lim_{z_0 \to 0} G_{1\uparrow}(z_0, z_1). \tag{62}$$

We shall set

$$V^{00}_{(1)} = V^{11}_{(1)} = 0, \; V^{00}_{(0)} = \frac{\pi}{2} r, \; V^{11}_{(0)} = \frac{\pi}{2} s,$$

which corresponds to an e-e interaction including only charge-density fluctuations (the usual TL model). Concerning the interaction between electrons and impurities, we shall take into account only spin-density and spin-current interactions,

$$U^{00}_{(0)} = U^{00}_{(1)} = 0, \; \left(U^{11}_{(0)}\right)^2 = \left(U^{11}_{(1)}\right)^2 = 2\pi^2 t.$$

Note that for repulsive electron-electron interactions one has $r > 0$ and $s > 0$, whereas $t > 0$ for both ferromagnetic and antiferromagnetic couplings.

Taking the limit $z_0 \to 0$ in (58) and replacing eqs. (58) and (59) in (62) one gets

$$N_{1\uparrow}(q) = C(\Lambda) \int dz_1 \frac{e^{-i(q-p_F)z_1}}{z_1} e^{-\int dp_1 \frac{1-cosp_1 z_1}{p_1} \Gamma(p_1)}, \tag{63}$$

where $C(\Lambda)$ is a normalization constant depending on an ultraviolet cutoff Λ, and $\Gamma(p_1)$ depends on p_1 through the potentials in the form

$$\begin{aligned} \Gamma(r,s,t) &= \frac{(\mid 1+r \mid^{1/2} -1)^2}{\mid 1+r \mid^{1/2}} + \frac{(\mid 1+s-t \mid^{1/2} - \mid 1-t \mid^{1/2})^2}{\mid 1+s-t \mid^{1/2} \mid 1-t \mid^{1/2}} - \\ &- \frac{2t(t-t_c)}{\mid 1+s-t \mid^{3/2} \mid 1-t \mid^{1/2}}. \end{aligned} \tag{64}$$

If we define

$$f(s,t) = \frac{(\mid 1+s-t \mid^{1/2} - \mid 1-t \mid^{1/2})^2}{\mid 1+s-t \mid^{1/2} \mid 1-t \mid^{1/2}}, \tag{65}$$

$$h(s,t) = t \frac{2t(t-1)+s(s-1)}{\mid 1+s-t \mid^{3/2} \mid 1-t \mid^{1/2}}, \tag{66}$$

we can write eq. (64) in the form

$$\Gamma(r,s,t) = f(r,0) + f(s,t) - h(s,t). \tag{67}$$

Once again, in order to go further and make the integration in (63), one has to specify the functional form of r, s and t. At this point one observes that we are in a position of discussing, through this simple example, an interesting aspect of the general model under consideration. Indeed, we can try to determine under which conditions it is possible to have a restoration of the Fermi edge. To this end, and as a first approximation, we shall consider contact interactions (r, s and t constants) and search for those relations between potentials giving $\Gamma(r,s,t) = 0$. In this last case one obtains the well-known normal Fermi-liquid (FL) behavior

$$N_{1\uparrow}(q) \approx \Theta(q - p_F). \tag{68}$$

At this point some remarks are in order. In the impurity free case ($t = 0$), $\Gamma(r,s,t)$ cannot vanish for any value of r and s other than $r = s = 0$, which corresponds to the non-interacting Fermi gas. This result is consistent with the well-known LL behavior of the TL model.

In order to have collective modes with real frecuencies ($\omega^2 > 0$), one finds two regions where the FL edge could be restored: $t > 1 + s$ and $t < 1$.
In eq.(65) one can observe that $f(r, 0) > 0$, thus setting $\Gamma = 0$ yields the condition

$$F(s, t) = h(s, t) - f(s, t) > 0. \tag{69}$$

A simple numerical analysis of $F(s, t)$ shows that the above inequality is not fulfilled for $0 < t < 1$. The electron-impurity coupling is not strong enough in this region as to eliminate the LL behavior. On the contrary, for $t > 1 + s$ equation (65) can be always satisfied. Moreover, in this region, we obtain a surface in which the condition $\Gamma = 0$ provides the following analytical solution for r in terms of $F(s, t)$

$$r = F^2/2 + 2F + (1 + F/2)(F^2 + 4F)^{1/2}. \tag{70}$$

The above discussion can be summarized by identifying the following three regions in the space of couplings:
Region I, given by $0 \leq t < 1$, in which one necesarilly has LL behavior. Region II, with $1 \leq t < s + 1$, in which the frecuency of the spin density excitations becomes imaginary; and region III, given by $t > s + 1$, where the FL behavior is admitted. In this region Eq(66) defines a surface in the space of potentials on which FL behavior takes place. One particular solution belonging to this surface is obtained by choosing $s = 0$ in (70), which yields

$$r(t) = \frac{2t(3t - 2) + 2(2t - 1)\sqrt{3t^2 - 2t}}{(t - 1)^2}.$$

corresponding to the case in which the dispersion relation of the spin density excitations is given by $\omega^2 = q^2$. For t large, r approaches a minimum value $r_{min} = 6 + 4\sqrt{3}$, a feature that is shared with each curve $s = constant$ on the "FL surface".

In summary, we have shown how the e-i couplings can be tuned in order to have a restoration of the Fermi edge in a TL model of electrons interacting with fermionic impurities. Unfortunately, we could analytically work out this mechanism only for a very peculiar choice of the e-i couplings, which evidently weakens its experimental relevancy. Besides, a more realistic study should include at least the backward-scattering processes. However, we think this discussion deserves attention as a first step towards a possible reconciliation between the standard TL model and the FL phenomenology.

5 Comments on further developments

I would like to end by making some remarks on recent works that help to improve our present understanding of the Thirring-like model discussed in this talk.

In ref.[3] we addressed our attention to the vacuum properties of the above mentioned model. As it is well-known, ground-state wave functionals (GSWF's) have in general very complex structures. Due to this fact, their universal behavior has been seldom explored in the past. Fortunately, in a recent series of papers, an alternative way to compute GSWF's was presented [21] [22] [23]. By conveniently combining the operational and functional approaches to quantum field theories, these authors provided a systematic path-integral method that, at least in the context of $1+1$ systems, seems to be more practical than the previously known semiclassical and Bethe ansatz techniques. We take advantage of these advances and applied them to shed some light on the vacuum structure of the NLT. In particular we got a closed formula that gives the probability of the vacuum state as a functional, not only of the density configuration but also of the potentials that bind the original fermionic particles of the system. This result allowed us to find a non-trivial symmetry of this vacuum with respect to the interchange of density-density and current-current potentials. Of course, this symmetry does not persist at the level of the dispersion relations of the collective modes (plasmons), to which the excited states are expected to contribute. We have also computed the general electromagnetic response of the model and the asymptotic behavior of GSWF's and density-waves frequencies for a wide variety of power-law potentials. This allowed us to identify different phases contained in the non-local Thirring model.

In ref.[4] we studied the non-local Thirring model with a relativistic fermion mass term included in the action. Performing a perturbative expansion in the mass parameter, we found that the NLT is equivalent to a purely bosonic action which is a simple non-local extension of the sine-Gordon model. Thus, we have generalized Coleman's equivalence [24] to the case in which the usual Thirring interaction is point-splitted through bilocal potentials. In the language of many-body, non-relativistic systems, the relativistic mass term can be shown to represent not an actual mass, but the introduction of backward-scattering effects [25]. Therefore, our result could provide an alternative route to explore the non-trivial dynamics of (gapped)

collective modes.

Finally, using the results of ref.[26] we have recently examined the finite-temperature extension of the NLT, which allowed us to discuss termodynamical and transport properties of the 1d electronic system [27].

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